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# Duality of a Supersymmetric Model with the Pati-Salam group

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**Abstract**

Recently one of the authors proposed a dual theory of a Supersymmetric Standard Model (SSM), in which it is naturally understood that at least one quark (the top quark) should be heavy, i.e., almost the same order as the weak scale, and the supersymmetric Higgs mass parameter $\mu$ can naturally be expected to be small. However, the model cannot have Yukawa couplings of the lepton sector. In this paper, we examine a dual theory of a Supersymmetric Model with the Pati-Salam gauge group $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ with respect to the gauge group $SU(4)_{PS}$. In this scenario, Yukawa couplings of the lepton sector can be induced. In this model the Pati-Salam breaking scale should be around the SUSY breaking scale.

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Recently, it has become clear that certain aspects of four dimensional supersymmetric field theories can be analyzed exactly [1, 2, 3, 4]. By using the innovation, it has been tried to build models in order to solve some phenomenological problems [4, 5, 6, 7]. One of the most interesting aspects is "duality" [1, 3]. By using "duality", we can infer the low energy effective theory of a strong coupling gauge theory. One of the authors suggested that nature may use this "duality". He discussed a duality of a Supersymmetric Standard Model(SSM). But unfortunately, his model does not have Yukawa couplings of the lepton sector. In this paper, we would like to discuss a duality of a supersymmetric(SUSY) model with Pati-Salam gauge group [8] in order to obtain all Yukawa couplings.

First we would like to review Seiberg's duality. Following his discussion [1], we examine $SU(N_C)$ SUSY QCD with $N_F$ flavors of chiral superfields,

| | $SU(N_C)$ | $SU(N_F)_L$ | $SU(N_F)_R$ | $U(1)_B$ | $U(1)_R$ |
|---|---|---|---|---|---|
| $Q^i$ | $N_C$ | $N_F$ | 1 | 1 | $(N_F-N_C)/N_F$ |
| $\bar{Q}_j$ | $\bar{N}_C$ | 1 | $\bar{N}_F$ | $-1$ | $(N_F-N_C)/N_F$ |

which has the global symmetry $SU(N_F)_L \times SU(N_F)_R \times U(1)_B \times U(1)_R$. In the following, we would like to take $N_F \geq N_C+2$, though in the case $N_F \leq N_C+1$ there are a lot of interesting features [3, 9, 10, 11]. Seiberg suggests [1] that in the case $N_F \geq N_C+2$ at the low energy scale the above theory is equivalent to the following $SU(\tilde{N}_C)$ SUSY QCD theory ($\tilde{N}_C = N_F - N_C$) with $N_F$ flavors of chiral superfields $q_i$ and $\bar{q}^j$ and meson fields $T^i_j$,

| | $SU(\tilde{N}_C)$ | $SU(N_F)_L$ | $SU(N_F)_R$ | $U(1)_B$ | $U(1)_R$ |
|---|---|---|---|---|---|
| $q_i$ | $\tilde{N}_C$ | $\bar{N}_F$ | 1 | $N_C/(N_F-N_C)$ | $N_C/N_F$ |
| $\bar{q}^j$ | $\bar{\tilde{N}}_C$ | 1 | $N_F$ | $-N_C/(N_F-N_C)$ | $N_C/N_F$ |
| $T^i_j$ | 1 | $N_F$ | $\bar{N}_F$ | 0 | $2(N_F-N_C)/N_F$ |

and with a superpotential

$$W = q_i T^i_j \bar{q}^j. \tag{1}$$

The above two theories satisfy the 't Hooft anomaly matching conditions [12]. Moreover Seiberg showed that they are consistent with the decoupling theorem [13]. Namely, if we introduce a mass term only for superfields $Q^{N_F}$ and $\bar{Q}_{N_F}$

$$W = mQ^{N_F}\bar{Q}_{N_F} \tag{2}$$

in the original theory, the dual theory has vacuum expectation value(VEVs) $\langle q \rangle = \langle \overline{q} \rangle = \sqrt{m}$ and $SU(N_f - N_c)$ is broken to $SU(N_f - N_c - 1)$, which is consistent with the decoupling of the heavy quark in the original theory.

Second, we would like to review a duality of a SUSY Standard Model(SSM) [5]. We introduce ordinary matter superfields

$$
\begin{aligned}
&Q_L^i = (U_L^i, D_L^i) : (3,2)_{\frac{1}{6}}, \quad U_{Ri}^c : (\bar{3},1)_{-\frac{2}{3}}, \quad D_{iR}^c : (\bar{3},1)_{\frac{1}{3}} \\
&L^i = (N_L^i, E_L^i) : (1,2)_{-\frac{1}{2}}, \quad E_{Ri}^c : (1,1)_1, \qquad i = 1,2,3,
\end{aligned} \tag{3}
$$

which transform under the gauge group $SU(3)_{\tilde{C}} \times SU(2)_L \times U(1)_Y$. There are no Higgs superfields. We would like to examine the dual theory of this theory with respect to the gauge group $SU(3)_{\tilde{C}}$. In the following, we neglect the lepton sector for simplicity. Since $N_F = 6$, the dual gauge group is also $SU(3)_C$ ( $\tilde{N}_C = N_F - N_C$ ), which we would like to assign to the QCD gauge group. A subgroup, $SU(2)_L \times U(1)_Y$, of the global symmetry group $SU(6)_L \times SU(6)_R \times U(1)_B \times U(1)_R$ is gauged. When we assign $Q = (U_L^1, D_L^1, U_L^2, D_L^2, U_L^3, D_L^3)$ and $\bar{Q} = (U_R^{c1}, D_R^{c1}, U_R^{c2}, D_R^{c2}, U_R^{c3}, D_R^{c3})$, the $SU(2)_L$ generators are given by

$$
I_L^a = I_{L1}^a + I_{L2}^a + I_{L3}^a, \quad a = 1,2,3, \tag{4}
$$

where $I_{Li}^a$ are generators of $SU(2)_{Li}$ symmetries which rotate $(U_L^i, D_L^i)$, and the generator of hypercharge $Y$ is given by

$$
Y = \frac{1}{6}B - (I_{R1}^3 + I_{R2}^3 + I_{R3}^3), \tag{5}
$$

where $I_{Ri}^a$ are generators of $SU(2)_{Ri}$ symmetries which rotate $(U_{Ri}^c, D_{Ri}^c)$. In this theory, the global symmetry group is $SU(3)_{QL} \times SU(3)_{UR} \times SU(3)_{DR} \times U(1)_B \times U(1)_R$. Then we can write down the quantum numbers of dual fields;

$$
\begin{aligned}
&q_{Li} = (d_{Li}, -u_{Li}) : (3,\bar{2})_{\frac{1}{6}}, \quad u_R^{ci} : (\bar{3},1)_{-\frac{2}{3}}, \quad d_R^{ci} : (\bar{3},1)_{\frac{1}{3}} \\
&M_j^i : (1,2)_{-\frac{1}{2}}, \quad N_j^i : (1,2)_{\frac{1}{2}}
\end{aligned} \tag{6}
$$

under the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Here $M_j^i \sim Q_L^i U_{Rj}^c$ and $N_j^i \sim Q_L^i D_{Rj}^c$ are the meson fields and we assign $q = (d_L^1, -u_L^1, d_L^2, -u_L^2, d_L^3, -u_L^3)$ and $\bar{q} = (d_R^{c1}, -u_R^{c1}, d_R^{c2}, -u_R^{c2}, d_R^{c3}, -u_R^{c3})$. It is interesting that the matter contents of both theories

are almost the same. The difference is the existence of nine pairs of Higgs superfields $M^i_j$ and $N^i_j$ and their Yukawa terms coupling to ordinary matter superfields,

$$W = -q^i_L N^j_i u^c_{Rj} + q^i_L M^j_i d^c_{Rj}. \tag{7}$$

It is interesting that the Yukawa couplings can be expected to be of order 1 because of the strong dynamics. This means that at least one quark has a heavy mass, which is almost the order of the weak scale $v$. It is also interesting that the SUSY mass terms of Higgs particles are forbidden by the global symmetry $SU(3)_Q \times SU(3)_{UR} \times SU(3)_{DR}$.

Unfortunately, the model has a lot of phenomenological problems. There is no Yukawa coupling of leptons, there appear Nambu-Goldstone bosons when the Higgs particles have vacuum expectation values, and the SUSY mass terms of Higgs particles vanish. Moreover we may give up the success of the unification of the three gauge couplings, because we cannot trace the running of the $SU(3)_{\tilde{C}}$ coupling.

In the following, we try to avoid the problem of Yukawa couplings of leptons and of the Nambu-Goldstone bosons. Why does not the above model have Yukawa couplings of leptons? This is because the leptons have no color charge. Therefore, we can expect that if we adopt the Pati-Salam gauge group $SU(4)_{PS}$ as the dual gauge group, the model has the Yukawa couplings of leptons.

We consider the dual gauge group of $SU(4)_{PS}$. The model which is extended minimally from MSSM has six flavors because a color triplet and a singlet belong to a quartet of $SU(4)_{PS}$. In this case, the dual group is $SU(2)$, and it is necessary to treat the model differently, since the model satisfies $N_F = 3N_C$ and is no longer in a non-abelian coulomb phase. The gauge coupling may not become strong. Therefore, we investigate another possibility of realizing $SU(4)_{PS}$. If we introduce fourth generation, we should impose some unnatural mass relations in order to suppress the Peskin-Takeuchi's S and T parameters [14]. Therefore, we would like to add one vector-like generation. Since $N_F$ becomes ten, the dual gauge group of the $SU(4)_{PS}$ becomes $SU(6)$. Here we introduce the following superfields:

$$\Psi^i_L : (6, \bar{2}, 1), \quad \Phi_L : (6, 1, \bar{2}), \quad \Psi^c_{Ri} : (\bar{6}, 1, 2), \quad \Phi^c_R : (\bar{6}, 2, 1), \qquad i = 1, 2, 3, 4, \tag{8}$$

under the gauge group $SU(6)_{HC} \times SU(2)_L \times SU(2)_R$. Since $N_F = 10$ under the gauge group $SU(6)_{HC}$, the dual gauge group becomes $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$. Then the

dual fields become

$$
\begin{aligned}
\psi_L^i : (4,2,1), \quad \phi_L : (4,1,2), \quad \psi_{Ri}^c : (\bar{4},1,\bar{2}), \quad \phi_R^c : (\bar{4},\bar{2},1),& \\
T_j^i : (1,\bar{2},2), \quad M_i^{(1)} : (1,1,1), \quad M_i^{(3)} : (1,3,1),& \\
N^{(1)i} : (1,1,1), \quad N^{(3)i} : (1,1,3), \quad S : (1,2,\bar{2}),&
\end{aligned} \tag{9}
$$

which transform under the gauge group $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$, and have the superpotential

$$
W = \psi_L^i T_i^j \psi_{Rj}^c + \psi_L^i (M_i^{(1)} + M_i^{(3)}) \phi_R^c + \phi_L (N^{(1)i} + N^{(3)i}) \psi_{Ri}^c + \phi_L S \phi_R^c. \tag{10}
$$

Namely this model has three generations and one vector-like generation with the Pati-Salam gauge group. You should notice that the Yukawa couplings of leptons appear. If we introduce soft SUSY breaking terms

$$
\begin{aligned}
\mathcal{L}_{SB}^e &= \sum_{i=1}^{4} \left( m_{\Psi Li}^2 |\Psi_L^i|^2 + m_{\Psi Ri}^2 |\Psi_{Ri}^c|^2 \right) + m_{\Phi L}^2 |\Phi_L|^2 + m_{\Phi R}^2 |\Phi_R^c|^2 \\
&\quad + \left( A_i \Psi_L^i \Phi_R^c + B^i \Phi_L \Psi_{Ri}^c + \frac{1}{2} \sum_{a=6,2l,2r} \mu_a \lambda_a \lambda_a + h.c. \right),
\end{aligned} \tag{11}
$$

in the original theory, all the global symmetries $SU(4)_L \times SU(4)_R \times [U(1)]^4$ except $U(1)_{B+L}$ can be broken explicitly. Therefore, we can avoid the massless Nambu-Goldstone bosons.

Here we only assume that the above duality can be realized even with the SUSY breaking terms. We can obtain the SUSY breaking terms of the dual theory,

$$
\begin{aligned}
\mathcal{L}_{SB}^e &= \sum_{i=1}^{4} \Big( m_{\psi Li}^2 |\psi_{Li}|^2 + m_{\psi Ri}^2 |\psi_R^{ci}|^2 + m_{M1i}^2 |M_i^{(1)}|^2 + m_{M3i}^2 |M_i^{(3)}|^2 \\
&\quad + m_{N1i}^2 |N^{(1)i}|^2 + m_{N3i}^2 |N^{(3)i}|^2 \Big) \\
&\quad + \sum_{i,j=1}^{4} \left( m_{Tij}^2 |T_j^i|^2 \right) + m_{\phi L}^2 |\phi_L|^2 + m_{\phi R}^2 |\phi_R^c|^2 + m_S^2 |S|^2 \\
&\quad + \left( \sum_{i=1}^{4} \left( A_i \mu N^{(1)i} + B^i \mu M_i^{(1)} \right) + \frac{1}{2} \sum_{a=4,2l,2r} \mu_a \lambda_a \lambda_a + h.c. \right),
\end{aligned} \tag{12}
$$

where we treat SUSY breaking parameters perturbatively [15] and $\mu$ denotes a typical scale of the dual dynamics. The scale $\mu$ should be larger than the SUSY breaking scale for the perturbation to be good approximation.[1] From the above SUSY breaking terms,

[1] Phenomenologically Higgsino masses, which are induced by the higher order of the perturbation, should be larger than the weak scale. In order to realize this situation, the scale $\mu$ cannot be much larger than the Pati-Salam scale.

we can find that the scalar fields $N_i^{(1)}$ and $M_i^{(1)}$ have vacuum expectation values(VEVs) of order $\mu$. Therefore, under the scale $\mu$ we will get the three family model with the Pati-Salam gauge group.

In this scenario, however, the Pati-Salam scale should be around the SUSY breaking scale because of the following two reasons. The first reason is that there is no vacuum which can break the gauge symmetry $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ to the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ in the flat direction. Namely, the Pati-Salam scale should be the order of the SUSY breaking scale for the potential problem. The second reason is that the Yukawa couplings of leptons become too small if the Pati-Salam scale is much larger than the weak scale. Namely since under the Pati-Salam scale, "leptons" in $\psi_L^i$ become massive with $T_i^4$ and ordinary leptons become three linear combinations of $T_j^i$, the Yukawa couplings of the lepton sector become very small.

It seems to be impossible for such a lower Pati-Salam scale to be consistent with the experimental bounds. From the bound [18] $\Gamma(K_L^0 \to e\mu)/\Gamma_{K_L^0} < 3.3 \times 10^{-11}$, the Pati-Salam scale is usually estimated to be larger than 1400 TeV [19]. However in the scenario where the $\tau$ lepton is associated with the down quark, the lower bound of the Pati-Salam scale becomes 13 TeV [19], which is not so far from the weak scale as 1400 TeV. Therefore, if the SUSY breaking scale is of order 10 TeV, we may satisfy the above experimental constraint. Though such a large SUSY breaking scale is unnatural, it can suppress flavor changing neutral currents. In future the signal of $B_S^0 \to \mu e$ may be found [19].

Though the structure of quark and lepton mass matrices are too complicated for us to analyze them, we should comment about the masses of neutrinos. In this model, two right-handed neutrinos have Majorana masses, which are order of the Pati-Salam scale, as a result of their mixing with gauginos( you should notice that the SUSY breaking scale is also of order the Pati-Salam scale ). Therefore, two left-handed neutrinos become light by seesaw mechanism. However, since one left-handed neutrino cannot use the seesaw mechanism, the Dirac masses of the left-handed neutrino should be less than of order 30 MeV, which is the experimental upper bound of the tau neutrino mass. In this case, other two neutrinos will be lighter than $(10\text{MeV})^2/(10\text{TeV}) \sim 10$ eV.

In summary, we examine duality of a SUSY model with the Pati-Salam gauge group. In this model, the Yukawa couplings of the lepton sector as well as of the quark sector can be induced. Since the Pati-Salam scale should be around SUSY breaking scale in this model, we should take the SUSY breaking scale larger than 10 TeV. The signal $B_S^0 \to \mu e$

may be found in future experiments.

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## References

[1] N. Seiberg, *Nucl. Phys.* **B435** (1995) 129; RU-95037, “The Power of Duality: Exact Results in 4-D SUSY Field Theory” (hep-th/9506077); D. Kutasov, *Phys. Lett.* **B351** (1995) 230; D. Kutasov and A. Shwimmer, *Phys. Lett.* **B354** (1995) 315; R.G. Leigh and M.J. Strassler, *Phys. Lett.* **B356** (1995) 492; K. Intriligator, R.G. Leigh, and M.J. Strassler, RU-95-38 (hep-th/9506148); P. Pouliot, RU-95-46 (hep-th/9507018).

[2] N. Seiberg, RU-94-64, “The Power of Holomorphy: Exact Results in 4-D SUSY Field Theory” (hep-th/9408013).

[3] K. Intriligator and N. Seiberg, *Nucl. Phys.* **B444** (1995) 125; RU-95-48, hep-th/9509066; K. Intriligator and P. Pouliot, *Phys. Lett.* **B353** (1995) 471.

[4] M. Dine, A.E. Nelson, Y. Nir, and Y. Shirman, SCIPP-95-32 (hep-ph/9507378); M. Dine, A.E. Nelson, and Y. Shirman, *Phys. Rev.* **D51** (1995) 1362; M. Dine and A.E. Nelson, *Phys. Rev.* **D48** (1993) 1277.

[5] N. Maekawa, KUNS-1361 (hep-ph/9509407).

[6] T. Hotta, K.I. Izawa, and T. Yanagida, UT-717 (hep-ph/9509201).

[7] M. J. Strassler, RU-95-69 (hep-ph/9510342).

[8] J. Pati and A. Salam, *Phys. Rev.* **D10** (1974) 275.

[9] I. Affleck, M. Dine, and N. Seiberg, *Nucl. Phys.* **B241** (1984) 493; **B256** (1985) 557.

[10] D. Amati, K. Konishi, Y. Meurice, G.C. Rossi, and G. Veneziano, *Phys. Rep.* **162** (1988) 169.

[11] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, *Nucl. Phys.* **B223** (1983) 445; **B260** (1985) 157; **B229** (1983) 381.

[12] G. 't Hooft, " Recent Developments in Gauge Theories", eds., G. 't Hooft (Plenum Press, New York, 1980).

[13] T. Appelquist and J. Carazzone, *Phys. Rev.* **D11** (1975) 2856; Y. Kazama and Y.P. Yao, *Phys. Rev. Lett.* **43** (1979) 1562; *Phys. Rev.* **D21** (1980) 1116; **21** (1980) 1138; **25** (1982) 1605.

[14] M. Peskin and T. Takeuchi, *Phys. Rev. Lett.* **65** (1990) 964; *Phys. Rev.* **D46** (1992) 381.

[15] O. Aharony, J. Sonnenschein, M. Peskin, and S. Yankielowicz, SLAC-PUB-95-6938 (hep-th/9507013).

[16] N. Evans, S.D.H. Hsu, M. Schwetz, and S.B. Selipsky, YCTP-P11-95 (hep-th/9508002); *Phys. Lett.* **B355** (1995) 475.

[17] I.I. Kogan, M. Shifman, and A. Vainshtein, TPI-MINN-95/18-T (hep-th/9507170).

[18] Particle Data Group, *Phys. Rev.* **D50** (1994).

[19] G. Valencia and S. Willenbrock, *Phys. Rev.* **D50** (1994) 6843.